

Exam. Code : 211002

Subject Code : 4276

M.Sc. (Mathematics) 2nd Semester

ALGEBRA-II

Paper-MATH-563

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Candidates are required to attempt **FIVE** questions, selecting at least **ONE** question from each section. The **fifth** question may be attempted from any section.

SECTION—A

1. (a) In a UFD, prove that a non-zero element is prime iff it is irreducible. 10
- (b) If R is commutative ring such that $R[x]$ is PID then prove that R is a field. 10
2. (a) Prove that R is UFD iff $R[x]$ is UFD. 10
- (b) State and prove Eisenstein's criteria for irreducibility of polynomials over \mathbb{Q} . 10

SECTION—B

3. (a) Prove that exists a splitting field for a polynomial $f(x) \in F[x]$. Further show that it is unique upto F -isomorphism. 10
- (b) Let $F \subseteq K \subseteq L$ be fields such that $[L:K]$ and $[K:F]$ is finite. Show that $[L:F]$ divides $[L:K]$. 10
4. (a) Prove that if F is a finite field then characteristic of F is prime number p and $|F| = p^n$ for some $n \geq 1$. 10

- (b) Prove that an irreducible polynomial over a field of characteristic 0 or over a finite field is separable. 10

SECTION—C

5. (a) Prove that for a given prime p and positive integer n there exists a field with p^n elements. Further, show that any two such fields are F -isomorphic. 10
- (b) Show that if F is a finite field of order p^n , then every element of F is p^{th} power in F . 10
6. (a) Prove that a polynomial is solvable by radicals if its Galois group is a solvable group. 10
- (b) Show that an extension of degree 2 is normal. Give an example of extension which is not normal. 10

SECTION—D

7. (a) Is every submodule of finitely generated free R -module free? Justify. 10
- (b) Show that if M is R -module and R is commutative ring then $\text{Hom}_R(M, M)$ is R -module. 10
8. (a) Let A, B be submodules of R -module M , prove that $A + B/B \cong A/A \cap B$. 10
- (b) Prove that a left R -module M is simple iff $M \cong \frac{R}{I}$,
 I maximal left ideal of R . 10